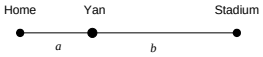
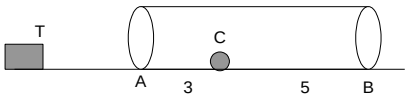
Traveling Problems in AMC

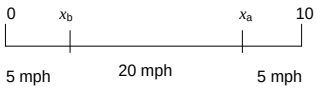
Problem	Solution
AMC 8 - Advanced George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first ½ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last ½ mile in order to arrive just as school begins today?	total time needed = $\frac{1}{3}$ hr $\frac{1/2}{2} = \frac{1}{4}$ hr $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ hr left $\frac{1/2}{1/12} = \frac{12}{2} =$ 6 mph
AMC 10 - Advanced Yan's home is 7 miles from the stadium. At the moment, Yan is somewhere between his home and the stadium. To reach the stadium, he has two options: - Walk directly to the stadium. - Walk back home and then ride his bicycle to the stadium. Yan rides his bicycle 7 times faster than he walks. Both options take the same amount of time. If Yan walks at a speed of 3 miles per hour, how long will it take him to stroll directly to the stadium?	Let a be the Yan's distance from home. Let b be the Yan's distance from stadium.  walking speed : bike speed = 1:7 $\frac{b}{1} = \frac{a}{1} + \frac{a+b}{7}$ $7b = 7a + a + b$ $6b = 8a$ $a:b$ = 3:4 b = 4 miles $\frac{4}{3} = 1\frac{1}{3} hrs$ = 80 min.
AMC 12 - Advanced A train is approaching a 1000-ft tunnel AB. Inside the tunnel, a cat is located at a point that is $\frac{3}{8}$ of the tunnel's length from the entrance A. When the train whistles, the cat starts moving. - If the cat runs toward the entrance A, the train reaches the entrance at the same time as the cat. - If the cat runs toward the exit B, the train reaches the exit at the same time as the cat. If the train moves at 60 miles per hour, how fast does the cat move in yards per minute? (1 mi = 5280 ft)	Let TA:AC = x : 3. When the cat moves leftward, the ratio of the train speed to the cat speed is x:3. (Why?) In the other direction, Time = $\frac{\text{distance}}{\text{speed}} = \frac{x+8}{x} = \frac{5}{3}$ $3x + 24 = 5x$ $2x = 24$ $x = 12$ TA:AC = 4:1 $1000 \times \frac{3}{8} \times 4 = 1500$ ft $60 \div 4 = 15$ (mph, cat's speed) $15 \times 5280 \div 3 \div 60 =$ 440 yd per min



AIME - Advanced

An e-bike travels at a speed of up to 20 mph and can carry one rider and one passenger simultaneously. The joggers move at a speed of 5 mph if without a ride. If Charles rides an e-bike to help Alex and Brian, two joggers, to finish a 10-mile trail, what is the minimum amount of time required for all three to finish the trail at the same time?

Method I)



Let x_a be the coordinate at which Alex gets dropped off.
Let x_b be the coordinate at which Brian gets picked up.

Time needed by Alex: $\frac{x_a}{20}+\frac{10-x_a}{5}$

Time needed by Brian: $\frac{x_b}{5}+\frac{10-x_b}{20}$

Time needed by Charles: $\frac{x_a}{20}+\frac{x_a-x_b}{20}+\frac{10-x_b}{20}=\frac{10+2(x_a-x_b)}{20}=\frac{5+(x_a-x_b)}{10}$

$$\frac{x_a}{20}+\frac{10-x_a}{5}=\frac{x_b}{5}+\frac{10-x_b}{20}$$

$$x_a+4(10-x_a)=4x_b+10-x_b$$

$$30=3(x_a+x_b)$$

$$x_a+x_b=10 \ldots ①$$

$$\frac{x_a}{20}+\frac{10-x_a}{5}=\frac{5+(x_a-x_b)}{10}$$

$$40-3x_a=10+2(x_a-x_b)$$

$$5x_a-2x_b=30 \ldots ②$$

2① + ②

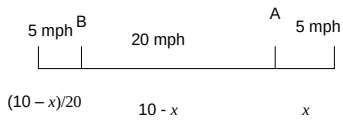
$7x_a = 50$

$$x_a=\frac{50}{7}$$

$$x_b=\frac{20}{7}$$

$T_a = T_b = T_c = \frac{40-3x_a}{20} = \frac{13}{14} = 0.9285$ hr

Method II)



$$T_1=\frac{10-x}{20}$$

$$T_a=\frac{x}{5}$$

	$D_2 = 10 - x - 5T_1 = \frac{3(10-x)}{4}$, the distance between Charles and Brian. $T_2 = \frac{D_2}{5+20} = \frac{3(10-x)}{100}$ Brian has jogged $T_1 + T_2 = \frac{2(10-x)}{25}$ hr, or $\frac{2(10-x)}{5}$ miles when he meets Charles. $T_1 + T_2 + \frac{10-5(T_1+T_2)}{20} = T_1 + \frac{x}{5}$ $T_2 + \frac{10-5(T_1+T_2)}{20} = \frac{x}{5}$ $\frac{3(10-x)}{100} + \frac{10-\frac{2(10-x)}{5}}{20} = \frac{x}{5}$ $3(10-x) + 50 - 2(10-x) = 20x$ $21x = 60$ $x = \frac{60}{21} = \frac{20}{7} < 3$ miles So, the total time = $\frac{10-\frac{20}{7}}{20} + \frac{\frac{20}{7}}{5} = \frac{5}{14} + \frac{4}{7} = \frac{13}{14} = \underline{0.9285}$ hr