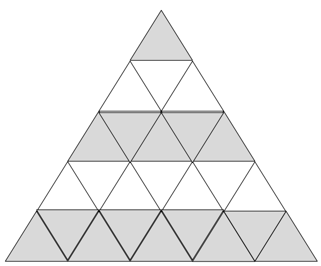
Answer Key

1. down:$5^3 = 125$
   across:$2^8 = 256$
   $3 + 8 = 11$

2. $2013 \div 3 = 671$
   $671 = 335+336$
   $671 = 334+337$
   $671 = 333+\underline{338}$

3. $100 = 4\times25$
   $25 = 1 + 24 = 1 + 8 + 16$
   $100 = 4\times25 = 2^2(1 + 2^3 + 2^4) = 2^2 + 2^5 + 2^6$
   3 of them:2^6, 2^5, and 2^2.
   Ans = 3

4. $1 + 3 + 5 + 7 + 9 = 5^2 = 25$
   $\frac{15}{25} = \frac{60}{100} = 60\%$



5. M:$\frac{2}{5}\times60 = 24$
   W:$\frac{2}{3}\times60 = 40$
   F:$\frac{2}{4}\times60 = 30$
   $24 + 40 + 30 = 94$ min
   $\frac{6}{4}\times60 = 90$ min
   $94 - 90 = 4$ min less

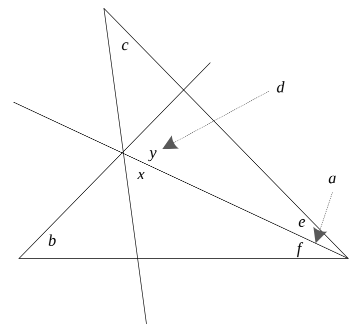
6. Let them be x, $x + 2$, $x + 4$, $x + 6$, and $x + 8$.
   $2x + 2 = x + 8$
   $x = 6$
   So, they are 6, 8, 10, 12, 14.
   $6+8+10+12+14 = 50$ cm

7. LCM(15, 4, 6, 12) = 60
   $60 = 1\times60 = 2\times30 = 3\times20 = 4\times15 = 5\times12 = 6\times10$
   60 has 12 factors.
   Ans = 60

8. The area becomes $\frac{1}{4}$ after two foldings.
   $9\times4 = 36$
   $36 = 6^2$
   $4\times6 = 24$

9. $3(36 + 44) + 10 = 250$

10. Draw an auxiliary line:
    $d = x + y$
    $a = e + f$
    $x = c + e$ (exterior angle theorem)
    $y = b + f$ (exterior angle theorem)
    $d = x + y = b + c + e + f$
    $= 55 + 40 + 35$
    $= 130$



11. $180 - 20 = 160$
    $\frac{1}{2}(160) = 80$ (two base angles)
    $\frac{1}{2}(80) = 40$ (angle bisected)
    $\angle CRT = 20 + 40 = 60$ (exterior angle theorem)

12. There are 2 pairs on each face.
    $6\times2 = 12$
    There are 2 pairs in space along each axes in 3-D.
    Just name one dimension along z-axis
    (perpendicular to the floor)
    (AD, FG) and (EH, BC).
    $3\times2 = 6$
    $12 + 6 = 18$ pairs in total

13. $4\times70 = 280$
    $3\times80 = 240$
    $280 - 240 = 40$

14. 4 terms
    $8 = 1 + 3 + 3 + 1$

15. C
    $706 \div 7 = 100R6$
    Saturday – 1 = Friday

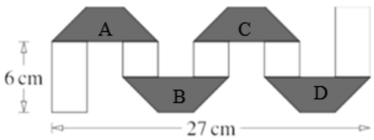
16. 10:20:30 = 1:2:3
    The weighted average
    $= \frac{1\times70\%+2\times80\%+90\times3}{1+2+3} = 83\%$

17. Let x be the number glasses.
    $\frac{1}{8}x = 3$
    $x = 24$
    $24 = 8 + 16$
    Let y be the number of guests.
    $\frac{1}{2}y = 8$
    $y = 16$

18. $\frac{1}{2}(50+10) = 30$
    $30^2 - 50\times10 = 400$

19. $3\times4+4 = 16$
$\frac{16}{36} = \frac{4}{9} = 4/9$

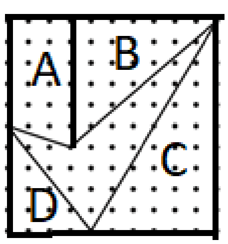
20. Method I)
$A = B = C = D = 9$
$6\times2 + 4\times9 + 3\times3 = 12 + 36 + 9 = \underline{57}$ cm



Method II)
The total area after folding
$= 2\times6\times3 + 4\times18 + 3\times9$
$= 36 + 72 + 27$
$= 135$
The area lost due to folding
$= 4\times9 = 36$
Area of the original strip
$= 135 + 36 = 171 = 3\times57$
The original length = 57 cm

21. A
$72 - 27 = 45$

22. $10\times10 = 100$
A: $\frac{1}{2}\times3\times(5 + 6) = 16.5$
B: $\frac{1}{2}\times6\times7 = 21$
C: $\frac{1}{2}\times6\times10 = 30$
D: $\frac{1}{2}\times4\times5 = 10$
$100 - (A + B + C + D) = 22.5$

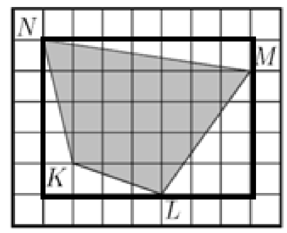


23. Let x be the side length of the smaller equilateral triangles.
$3(6 - 2x) + 3x = 18 - 3x = 9x$
$12x = 18$
$x = 1.5$ cm

24. 2 different values
$1+2+3+4+7 = 1+2+3+5+6 = 17$
$1\times2\times3\times4\times7 = 168$
$1\times2\times3\times5\times6 = 180$

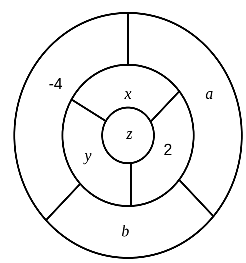
25. 5 of them:
9, <u>10</u>, 11
23, <u>24</u>, 25
47, <u>48</u>, 49
79, <u>80</u>, 81
81, <u>82</u>, 83

26. $5\times7 = 35$
$35 - \frac{1}{2}(1\times7 + 4\times3 + 3\times1 + 4\times1) - 1$
$= 35 - 14$
$= 21\text{ cm}^2$



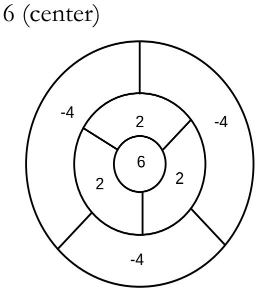
27. B
Cut it horizontally and vertically by some numbers:
7 by 0:# 8 pcs
6 by 1:# $7\times2 = 14$ pcs
5 by 2:# $6\times3 = 18$ pcs
4 by 3:# $5\times4 = 20$ pcs

28. $-4 = a + b + x + y$
$2 = z + \underline{a + b + x + y}$
$2 = z - 4$
$z = 6$



BTW:
$a = b + x - 2$
$b = a + y - 2$
$y = b + x + 4$
$x = a + y + 4$

$x + y = 4$
$a + b = -8$
$z = 6$ (center)



29. $1+3 = 4 = 2^2$
$1+3+5 = 3^2$
$1+3+5+\ldots+39 = 20^2 = 400$

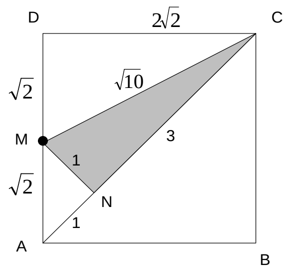
30. $\frac{1}{2}(6\times7) = 21$

31. width:$3\times16 = 18$
height:$3\times7 = 21$
$18+21 = 3(6+7) = 39$

32. $\frac{1}{2}(18\times21) = 189$

33. $6\times6 = 36$
Each side is 6.
The total area of shaded region
$= \frac{1}{2}\times6\times(p+q+r+s) = 27$
$p+q+r+s = 9$

34. Let MN = 1.
The area of the shaded region:$\frac{3}{2}$.
The area of the square:8.
Ans = 3:16



35. $p = 2, q = 5, r = 4$
$2\times5\times4 = 40$

36. 2 such numbers
10 and 20.
For 10, the only 3-multiples are {3, 6, 9}, so $\frac{3}{10} =$ 30%.
For 20, the only 3-multiples are {3, 6, 9, 12, 15, 18}, so $\frac{6}{20} =$ 30%.
For 30, the only 3-multiples are {3, 6, …, 27, 30}, so $\frac{10}{30} = 33\frac{1}{3}$%.
For 40, the only 3-multiples are {3, 6, …, 39}, so $\frac{13}{40} =$ 32.5%.
For 50, the only 3-multiples are {3, 6, …, 48}, so $\frac{16}{50} =$ 32%.
…

37. 4 EUR

38. $\frac{3}{1}-\frac{4}{2}=1$
$\frac{4}{1}-\frac{3}{2} = 2.5$

39. Let n coins be dug up by x men.
$\frac{n}{x-4}-\frac{n}{x}=10$
$4n = 10x(x-4)$

$\frac{n}{x}-\frac{n-50}{x}=5$
$50 = 5x$
$x = 10$
$n = 600\div4 = 150$

40. $1-\frac{2}{3} = \frac{1}{3}$
$\frac{3}{4} - \frac{1}{3} = \frac{5}{12} = 5/12$

41. Both got green:
$\frac{1}{2}\times\frac{1}{4} = \frac{1}{8}$
Both got red:
$\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$
$\frac{1}{8} + \frac{1}{4} = \frac{3}{8} = 3/8$

42. Method I) Data

Number	Row	Col	Index
1	1	1	1st
3	1	3	2nd
6	1	6	3rd
10	2	2	4th
15	2	7	5th
21	3	5	6th
28	4	4	7th
36	5	4	8th
45	6	5	9th
55	7	7	10th
66	9	2	11th
78	10	6	12th
91	12	3	13rd
105	14	1	14th
120	16	**0**	15th
136	18	0	16th
153	20	1	17th

Track these shaded number on the first column of the above table.
The column of each such number x = x mod 8
The answer is 120.

Method II) Algebra
Find the triangular number
T = 1+2+3 + … + n , and
T mod 8 = 0
Namely,
$8 \mid \frac{1}{2}n(n+1)$ or 8 is a divisor of $\frac{1}{2}n(n+1)$
Let's try
$n = 7$, T = 28, not good.
$n = 8$, T = 36, not good
$n = 15$, T = 120
GOOD, indeed an 8-multiple.

43. $\frac{30+5}{60} = \frac{7}{12} = 7/12$

44. Assume the number reads:
ab
$a + b + ab = 10a + b$
$a(1+b) = 10a$
$1+b = 10$
$b = 9$

45. $\frac{1}{3} = 1/3$

Bottom	Product of the remaining 5 face values
1	720
2	360
3	240
4	180
5	144
6	120

46. 6

A	B	C	Total
x	y	36	$x+y+36$
x-y-36	$2y$	72	$x+y+36$
$2x$-$2y$-72	$3y$-x-36	144	$x+y+36$
Not needed	Not needed	252-x-y =36	

47. $252 - x - y = 36$
$x + y + 36 = 252$

48. 2+3+4=1+3+5=1+2+6=9
2×3×4 = 24

49. 180÷3 = 60
60 + 60 = 120 (not the best)
180 ÷ 2 = 90
0, 90, 90 ⇒ 0 + 90 = 90 (but not valid)
1, 89, 90 ⇒ 1 + 90 = 91

50. $\frac{1}{4}$= 1/4

51. $a + b + y + z = \frac{1}{2}S$
$a + b = \frac{1}{3}S$
$y + z = \frac{1}{6}S.$
$\square = \frac{1}{6} = 1/6$

52. $y + z + w + x = \frac{1}{2}S$
$x = (\frac{1}{2} - \frac{1}{6} - \frac{1}{4}) = \frac{1}{12} = 1/12$

53. $2 * \square = 1$
$6 - \square = 1$
$\square = 5$
$5 * \Delta = 5$
$15 - \Delta = 5$
$\Delta = 10$

54. D
456<u>231</u> should be corrected as
456<u>321</u>

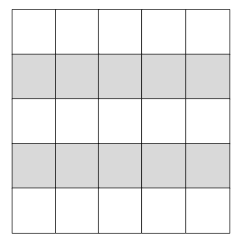
55. E

	Cor	Unan	Incorr	
90	18	0	2	20
91	18	1	1	20
92	18	2	0	20
95	19	0	1	20
97	**19**	**2**	**0**	**21**

56. 1357 + 2468 = 3825

57. E
3¾ - 2¼ = $1\frac{1}{2}$ (counter)
So, it points to east now

58. 16
The following figure only shows 15 empty cells.
Can you find a way to create 16 empty cells?



59. 0
$\frac{1}{2^2} = \frac{1}{4}$
$\frac{1+2}{3^2} = \frac{1}{3}$
$\frac{1+2+3}{4^2} = \frac{6}{16} = \frac{3}{8}$
…
$\frac{1+2+\cdots+7}{8^2} = \frac{28}{64} = \frac{7}{16}$

60. The leading digit must be non-zero, so there 9 choices.
The second digit must be different from the first one, so there are 9 choices.
The third one, 8 choices
The fourth one, 7 choices.
9×9×8×7 = 4536