Answer Key

1. N/A
2. N/A
3. N/A
4. N/A
5. N/A
6. N/A
7. N/A
8. N/A
9. N/A
10. N/A
11. 1
12. 1
13. 1
14. 1/4
15. 3/4
16. 2/3
17. 1
18. 1
19. 1/3
20. 1/6
21. $\frac{2}{6}$
22. $\frac{2}{6}$
23. $\frac{3}{7}$
24. $\frac{3}{6}$
25. $\frac{3}{6}$
26. $\frac{4}{7}$
27. $\frac{5}{8}$
28. $\frac{4}{7}$
29. $\frac{3}{6}$
30. $\frac{2}{5}$
31. 16
32. 12
33. 13
34. 22
35. 11
36. 24
37. 9
38. 20
39. 16
40. 15
41. -
42. 4.8
43. 3.4
44. 3.5
45. 5.6
46. 8.2
47. 3.3
48. -
49. 8.1
50. 1.9
51. 9
52. 12
53. 18
54. 15
55. 20
56. 6
57. 34
58. 27
59. 17
60. 6
61. 20
62. 29
63. 32
64. 29
65. 28
66. 13
67. 22
68. 15
69. 6
70. 11
71. A=4, B=7, C=5
    Ans=4 (for A) & 7 (for B) & 5 (for C)
72. A=6, B=9, C=9
    Ans=6 (for A) & 9 (for B) & 9 (for C)
73. A=1, B=9, C=0
    Ans=1 (for A) & 9 (for B) & 0 (for C)
74. A=9, B=2, C=1
    Ans=9 (for A) & 2 (for B) & 1 (for C)
75. A=3, B=6, C=1
    Ans=3 (for A) & 6 (for B) & 1 (for C)
76. A=2, B=4, C=2
    Ans=2 (for A) & 4 (for B) & 2 (for C)
77. A=5, B=7, C=3
    Ans=5 (for A) & 7 (for B) & 3 (for C)

78. A=1, B=6, C=1
    Ans=1 (for A) & 6 (for B) & 1 (for C)
79. A=5, B=6, C=6
    Ans=5 (for A) & 6 (for B) & 6 (for C)
80. A=1, B=3, C=1
    Ans=1 (for A) & 3 (for B) & 1 (for C)
81. 24×3 = 72 min
82. 24÷3 = 8
83. 27 + 35 = 62
84. 100 – 90 = 10
85. 5×40 = 200
86. 2×35 = 70
    100 – 70 = 30 cts
87. 62 – 16 = 46
88. 8 buffalo
89. 35 - 6 = 29
90. 16 + 13 = 29
91. 15 + 17 = 32
92. 25 + 12 + 23 = 60
93. 3+2+4 = 9
94. 3×15 = 45
    2×14 = 28
    2×12 = 24
    45 + 28 + 24 = 97 eggs
95. 4×4 = 16
    16 + 1 = 17
    Ans = 4 boxes
96. 4×5 = 20
    20 - 17 = 3 cones left over
97. 3×2 + 1.5 + 3×3
    = 6 + 1.5 + 9
    = $16.50
98. 20+30+40 = 90
99. C
100. 3×20 = 60

Answer Key

1. 60
2. 6
3. 3
4. 15
5. 7
6. 30
7. 4
8. 18
9. 8
10. 40
11. 1.75 = $1\frac{3}{4}$ = 1 3/4
12. $\frac{17}{5}$ = 17/5
13. $6\frac{1}{3}$ = 6 1/3
14. 2.75
15. $2\frac{7}{20}$ = 2 7/20
16. 2.4
17. $6\frac{1}{6}$ = 6 1/6
18. 20.2
19. $3\frac{3}{4}$ = 3 3/4
20. $6\frac{2}{5}$ = 6 2/5
21. $\frac{13}{20}$ = 13/20
22. $\frac{11}{20}$ = 11/20
23. 8.55
24. 3.25
25. 0.85
26. $\frac{9}{20}$ = 9/20
27. $6\frac{3}{14}$ = 6 3/14
28. $7\frac{1}{2}$ = 7 1/2
29. $\frac{3}{20}$ = 3/20
30. $\frac{16}{25}$ = 16/25
31. 0.5
32. $\frac{1}{5}$ = 1/5
33. 0.1
34. $\frac{2}{5}$ = 2/5
35. 0.75
36. $\frac{1}{2}$ = 1/2
37. 0.25
38. $\frac{3}{5}$ = 3/5
39. 0.25
40. $\frac{4}{5}$ = 4/5
41. 0.8
42. $\frac{1}{4}$ = 1/4
43. 0.2
44. $\frac{3}{4}$ = 3/4
45. 0.8
46. $\frac{3}{20}$ = 3/20
47. 1.25
48. $\frac{9}{20}$ = 9/20
49. 2.75
50. $\frac{17}{20}$ = 17/20
51. 6
52. 27
53. 65
54. 57
55. 8
56. 6
57. 1
58. 9
59. 4
60. 9
61. 13 & 3 (Remainder)
62. 104
63. 17.6
64. 13.8
65. 11
66. 228
67. 3
68. 9
69. 126
70. 7.6
71. 5/13
72. 4 3/10 – 2 1/10 = 2 1/5
73. 42 + 12 = 54 min
74. 4
75. 1 ⅓ + ⅜ + ¾ = 2 11/24
76. 4
77. 6×12.5 = $75

MAP 235 (T3) Issue 2

78. $20 \div \frac{1}{2} = 40$

79. $4 \div \frac{1}{12} = 48$

80. 91.1 – 70.2 = <u>20.9 pounds</u>

81. 2×6 = 12
    2×8 = 16 (pieces)
    $\frac{3}{16} \times 12 = \frac{9}{4} = \$2.25$

82. ⅓ + 1/9 = 4/9
    1 – 4/9 = 5/9

83. 5 ¼ - 3 ⅔ = 1 7/12

84. 40 + 15 = 55 min

85. 20

86. $3 \times 2\frac{3}{4} = 8\frac{1}{4} = 8$ 1/4 mi

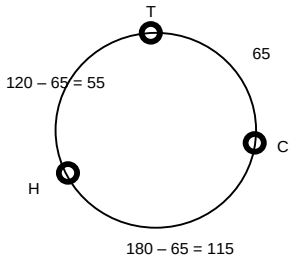
87. $1 - \frac{1}{3} = \frac{2}{3}$
    $24 \times \frac{2}{3} =$ <u>16</u>

88. $8 \div \frac{2}{3} = 12$

89. $1 - \frac{1}{3} = \frac{2}{3}$
    $27 \times \frac{2}{3} = 18$
    $1 - \frac{2}{3} = \frac{1}{3}$
    $18 \times \frac{1}{3} = 6$

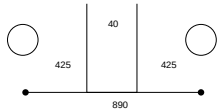
90. ⅓ = 3/9
    Original = 4/9

91. 180 – 65 = 115
    Another route: 120
    Ans= 115 the shortest one



92. □ – 6 = 2 + ½□
    ½□ = 8
    □ = 16

93. 891 – 41 = 850
    850÷2 = 425



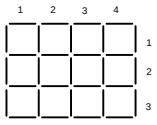
94. 13 + 5 = 18
    18÷3 = 6

95. The perimeter remains unchanged with the 3 cut-off corners.
    Ans = 30 cm

96. 192 - 2×84 = 24
    84 - 3×24 = 12
    Or, GCD(192, 84) = 12 *mm*
    This is the origin of <u>Euclidean algorithm</u>.

97.
$$\begin{array}{cccc} & 6 & 4 & 2 \\ + & 5 & 3 & 1 \\ \hline 1 & 1 & 7 & 3 \end{array}$$
    Ans = 1173

98. 5

99. 
    4×4 = 16 (horizontal cuts)
    3×5 = 15 (vertical cuts)
    Total: 16 + 15 = 31

100. 2 × 3 × 8 × 3
     = 4×3×4×3
     = 12×12
     ✷ = 12

Answer Key

1. 350
2. 6.4
3. 4.8
4. 4.5
5. 2.8
6. 560
7. 81
8. 630
9. 45
10. 240
11. 228
12. 143
13. 168
14. 195
15. 224
16. 255
17. 288
18. 323
19. 187
20. 216
21. 0.0125
22. 6
23. 0.0375
24. 400
25. 1.5
26. 0.05
27. 7500
28. 50
29. 0.3
30. 0.001
31. 4
32. 8
33. 3
34. 9
35. 2
36. 10
37. 16
38. 20
39. 15
40. 21
41. 128 (A) & 8 (B) & 8 (C) & 8 (D)
42. 42 (A) & 7 (B) & 7 (C) & 2 (D)
43. 72 (A) & 8 (B) & 8 (C) & 3 (D)
44. 168 (A) & 7 (B) & 4 (C) & 6 (D)
45. 175 (A) & 5 (B) & 3 (C) & 7 (D)
46. $\frac{41}{75}$
GCF = 5, LCM = 75
47. $\frac{11}{30}$
GCF = 5, LCM = 30
48. $\frac{3}{10}$
GCF = 5, LCM = 10
49. $\frac{3}{10}$
GCF = 15, LCM = 30
50. $\frac{41}{45}$
GCF = 3, LCM = 45
51. $\frac{46}{275}$
GCF = 5, LCM = 275
52. $\frac{7}{12}$
GCF = 2, LCM = 12
53. $\frac{11}{24}$
GCF = 2, LCM = 24
54. $\frac{5}{72}$
GCF = 3, LCM = 72
55. $\frac{9}{20}$
GCF = 2, LCM = 20
56. 9
57. 8
58. 4
59. 3
60. 5
61. 32
62. 8
63. 3
64. 6
65. 2
66. 2 in (A) & 3 in (B) & 2 in (C) & 5 in (D)
67. 2 in (A) & 5 in (B) & 4 in (C) & 3 in (D)
68. 3 in (A) & 2 in (B) & 8 in (C) & 3 in (D)
69. 5 in (A) & 3 in (B) & 6 in (C) & 5 in (D)
70. 4 in (A) & 3 in (B) & 5 in (C) & 4 in (D)
71. $32.5\times(17 + 13) = 32.5\times30 = 325\times3 = 975$
72. $144\pi = 12^2\pi$
$2\times12\pi = 24\pi = 24\ \pi$ inches

MAP 255 (T3) Issue 2

73. 25 - 4 = 21
    21÷25 = 84%
74. 12 for $2
    6 for $1
    18 for $3
    18 - 12 = 6 oranges added
75. 120:144 = 10:12 = 5:6
76. 30÷5% = 30 ÷ 0.05 = 3000 ÷ 5 = $600
77. 8 + 9 + 4 = 21
    See the steps below.

    21 - (9 + 5) = 7

8		
9	7	5
4	*n*	

    21 - (4 + 7) = 10

8		10
9	7	5
4	*n*	

    21 - (8 + 10) = 3

8	3	10
9	7	5
4	*n*	

    21 - (3 + 7) = 11

8	3	10
9	7	5
4	**11**	

78. LCM(6, 10) = 30
    LCM(30, 8) = 120
79. E
80. 2(80 + 60) = 280
    280÷10 = 28 posts
81. C
    For A: 12÷20=0.6
    For B: 18÷30=0.6
    Thus, both have the same steepness.
82. 25%×$600 = $150
83. 600 + 150 = $750
84. 10 matches needed
    RY, RG, RB, RW,
    YG, YB, YW,
    GB, GW, and
    BW.
85. ① 9+8+7+65+4+3+2+1 = 99, 7 addition signs.
    ② 9+8+7+6+5+43+21 = 99, 6 addition signs.
    Ans = 6

Answer Key

1. -13
2. -20
3. -12
4. -10
5. -6
6. -1
7. -4
8. 3
9. 1
10. 2
11. $-5x - 9$
12. $4x + 2$
13. $8x + 5$
14. $4x + 17$
15. $-10x$
16. $-9x - 18$
17. $-8x + 3$
18. $6x - 8$
19. $3x - 9$
20. $-4x + 8$
21. 42
22. -3
23. 99
24. -40
25. 15
26. -9
27. 20
28. 16
29. 33
30. 20
31. 1/2
32. 1/2
33. 256
34. 1/2
35. 1/2
36. 9
37. 1/2
38. 1/2
39. 11
40. 1/2
41. 90
42. 8/125
43. 27/81
44. 1/2
45. 1/4
46. 2
47. 7
48. 9
49. 4
50. 1/128
51. -1
52. -1
53. 9/4
54. 4/5
55. 5
56. 4
57. 7/3
58. 17/2
59. 5/4
60. 6/5
61. $\frac{5-4}{5} = \frac{1}{4} = 0.25 = 25\%$
62. $\frac{60-48}{60} = 0.2 = 20\%$
63. 12:18 = 2:3
    So, the increase is 50%.
64. 800:600 = 4:3
    $\frac{1}{4} = 25\%$
65. 80:64 = 5:4
    $\frac{1}{5} = 0.2 = 20\%$
66. $12 \div 30\% = 12 \div 0.3 = \boxed{40}$
67. 40 - 12 = 28
    $12 \times \frac{7}{3} = \boxed{28}$
68. $\frac{2}{5} = 0.4 = 40\%$
69. $2{,}000 \times 0.8 = \$1{,}600$
70. 1 + 20% = 1.2
    $20 \times 1.2 = \$24.00$ (new price)
    or
    $20 \times 20\% = 20 \times 0.2 = 4$
    20 + 4 = \$24.00
71. $15\% \times 20 = 0.15 \times 20 = 1.5 \times 2 = \$3$
72. $1 + \frac{1}{4} = \frac{5}{4}$
    $20 \times \frac{5}{4} = 25$
    or
    $20 \times \frac{1}{4} = 5$
    20 + 5 = 25
73. $\frac{discount}{original\ price} = \frac{10}{25} = 40\%$

74. 35÷5 = 7
    7×7 = 49

75. 35÷7 = 5
    5×5 = 25

76. 5 + 7 = 12
    60 ÷12 = 5
    5×5 = 25 (black)
    5×7 = 35 (red)

77. C
    Alex: $\frac{36}{12}$ = 3 pages per min
    Ben: $\frac{45}{15}$ = 3 pages per min

78. 10-12: 84 + 73 + 118 = 275
    7 – 9: 70 + 101 + 29 = 200
    275:200 = 11:8

79. D

80. B
    Test 1:83.3%
    Test 2:85.7%

81. 35÷(2+5) = 5
    2×5 = 10 gallons = 40 quarts

82. $\frac{\Delta}{4} = \frac{12}{6} = \frac{2}{1}$
    Δ = 8

83. 3.0÷12 = \$0.25 per ounce (12-ounce cheaper)
    5.4÷18 = \$0.30 per ounce

84. 96÷6 = 16 (miles per gallon)
    240÷16 = 15 gallons

85. A
    30°-60°-90°

86. D
    1 + 1 + 2 = 4
    180°÷4 = 45
    45°, 45°, and 90°

87. A
    $m + n + q = (2 + 1 + 3)n = 180$
    $n = 30$, $m = 60$, $q = 90$
    Right triangle

88. $\frac{2}{5}$×20 = \$8.00

89. Josh: 1
    David: 2
    Jennifer: 2×2 = 4
    1 + 2 + 4 = 7
    280÷7 = 40
    40×1 = 40 (Josh)
    40×2 = 80 (David)
    40×4 = 160 (Jennifer)

90. 10÷2 = 5
    5×16 = 80 bottles

91. Alex has \$50, therefore, Ben has \$100 (since their ratio is 1:2). Thus, Carl has \$75.

92. 36÷2 = 18 (half-perimeter)
    18 - 10 = 8 in (width)

93. 1-BR : x
    3-BR : $x + 25$
    2-BR : $3x – 15$
    $x + 3x - 15 + x + 25 = 260$
    $5x + 10 = 260$
    1-BR = $x = 50$
    2-BR = 135
    3-BR = 75
    Ans = 50 (1-br) & 135 (2-br) & 75 (3-br)

94. $100 \div (1\frac{1}{4}) \times 1.75$
    $= 100 \div \frac{5}{4} \times \frac{7}{4}$
    $= 100 \times \frac{4}{5} \times \frac{7}{4}$
    = 140

95. Method I)
    N : C : T
    = 3 : 1 : 4
    = 21 : 7 : 28

    Method II)
    Let r be the amount of cardboard in tons recycled each week. Then, the newspaper has $3r$ tons for recycle. The total is
    $4r = 28$
    $r = 7$
    Ans = 7 tons of cardboard & 21 tons of newspaper

96. Let x represent the number sought.
    $8x + 3x = 20 + 9x$
    $2x = 20$
    $x = 10$

97. B
    776+393 + 1 = 1170
    1170÷4 = 292R2
    292 + 1 = 293

98. If all of them are quarters, the total value would be 30×\$.25 = \$7.50. There is a slack of 7.50 - 4.10 = \$3.4. The difference of the value of a quarter and a nickel is 20¢, there are 340÷20 = 17 nickels and 30 - 17 = 13 quarters.
    Ans = 17 nickels & 13 quarters

99. Let x pennies be the price of the pen.
    $(x - 24) + (x - 4) < x$
    $x < 28$
    Ans = 27¢

100. 3 different ways:

101. Let s be the son's age. Then, the father is $4s$. The second statement is expressed as the following equation:

$$4s+3 = 1+3(s + 3)$$

Solving the equation the son's current age is $s = 7$ and the father's current age is $4s = 28$.

102. Method I)

P (1¢)	N (5¢)	D (10¢)	Q (25¢)	Value
13	26			143
15	24			135
20	19			115
25	14			95
25	**13**	**1**		100
30	9			75
30	**7**	**1**	**1**	100
35	**1**	**1**	**2**	100

Method II)
The number of pennies must be a multiple of 5 since the value of nickel, dime or quarter is so. Thus, we only need to consider 15, 20, 25 pennies, etc.
Say if 15 pennies are used, then 24 other coins are used. Can we make 85 pennies with 24 coins in nickel, dime or quarter? To put it in algebra, can we solve the following system of linear equations ?

$n + d + q = 24$
$5n + 10d + 25q = 85$
or
$n + 2d + 5q = 17$
The answer is negative. (Why ?)

Say if 20 pennies are used, then 19 other coins are used. We end up with solving the system of linear equations:

$n + d + q = 19$
$n + 2d + 5q = 16$
Again, no solution can be found.

However, we will find the solution if 25 pennies are used, or 14 other coins are used.

$n + d + q = 14$
$n + 2d + 5q = 15$
$p = 25$, $n = 13$, $d = 1$
Ans = 25 P & 13 N & 1 D

Answer Key

1. $\frac{1}{7}$
2. $4^{3x} = (4^x)^3 = 7^3$
3. $2^x = \sqrt{4^x} = \sqrt{7}$
4. $8^{-2x} = 2^{-6x} = (4^x)^{-3} = \frac{1}{7^3}$
5. $\frac{1}{16^x} = 16^{-x} = 4^{-2x} = (4^x)^{-2} = \frac{1}{7^2}$
6. 3
7. 2
8. 8
9. a) R4
   b) R1
   c) R3
   d) None
   e) R2
10. 3 (R3)
11. -3 (R1)
12. $\frac{1}{3}$ (R4)
13. $-\frac{1}{3}$
14. 2
15. -1
16. -2
17. 4
18. 5
19. -3
20. -3
21. 16
22. ½
23. $x = ab$
    Choice B reflects this relation perfectly.
24. $x = cd$
    Choice C reflects this relation perfectly.
25. $x = y_{\tilde{}}$
26. $\log_a \boldsymbol{c}$
27. 5
28. Method I)
    Let $x = \log_a b \log_b c$
    $a^x = (a^{\log_a b})^{\log_b c} = b^{\log_b c} = c$
    $x = \log_a c$

    Method II)
    $b = a^x$
    $c = b^y = (a^x)^y = a^{xy}$
    $\log_a c = xy = \log_a b \log_b c$
29. $\log_2 4 = 2$
30. $\log_3 27 = 3$
31. $\log_{10}11 \log_{11}12 \log_{12}13 \ldots \log_{999}1000$
    $= \frac{log\,11}{log\,10} \frac{\log 12}{\log 11} \frac{log\,13}{log\,12} \cdots \frac{log\,1000}{log\,999}$
    $= 3$
32. Let $a = c^x$
    Let $b = c^y = (c^x)^{y/x} = a^{y/x}$
    Therefore, $\log_a b = y/x = \frac{\log_c b}{\log_c a}$
33. 3
34. 2
35. 5
36. 3/2
37. 4/3
38. 3/2
39. -3/2
40. -4
41. log 4/log 2 = 2log 2/log 2 = 2
42. log 8/log 2 = 3log 2/log 2 = 3
43. log ¼/log 2 = -log 4/log 2 = -2
44. log ⅛/log 2 = -log 8/log 2 = -3
45. 6
46. $\frac{a+b}{ab}$
47. $\frac{ab}{a+b}$
48. $\log ab$
49. $\frac{\log ab}{\log a \log b}$
50. $\frac{\log a \log b}{\log ab}$
51. $\frac{-\log a \log b}{\log ab}$
52. Yes.
53. False
54. log 5 = log 10/2 = 1 - log 2 ≈ 0.699
55. $\log \frac{8}{81}$ = log 8 - log 81 = 3log 2 - 4log 3 ≈ 3×0.301 - 4×0.477 = -1.005
56. log 60 = log 6×10 = log 6 + 1 = log 2 + log 3 + 1 = 1.778
57. log 13500 = log 27×5×100 = log 3^3 + log 5 + log 100 = 3log 3 + log 5 + 2 ≈ = 3×0.477 + 0.699 + 2 = 4.13
58. $\log \sqrt{270} = \frac{1}{2} \log 270 = \frac{1}{2}(\log 27 + \log 10) = \frac{1}{2}(3\log 3 + 1) \approx \frac{1}{2}(3\times0.477 + 1) \approx 1.216$
59. $\log \sqrt[3]{\frac{9}{32}} = \frac{1}{3}(\log 9 - \log 32) = \frac{1}{3}(2\log 3 - 5\log 2) \approx \frac{1}{3}(2\times.477 - 5\times.301) = -0.184$

60. $\log_{12}27 = \log_{12}3^3 = 3\log_{12}3 = a$
$\log_{12}3 = \frac{a}{3}$
$\log_3 12 = \frac{3}{a}$
$1 + 2\log_3 2 = \frac{3}{a}$
$2\log_3 2 = \frac{3}{a} - 1 = \frac{3-a}{a}$
$\log_3 2 = \frac{3-a}{2a}$
$\log_2 3 = \frac{2a}{3-a}$
$\log_6 16 = 4\log_6 2$
$\log_2 6 = 1 + \log_2 3 = 1 + \frac{2a}{3-a} = \frac{3+a}{3-a}$
$\log_6 2 = \frac{3-a}{3+a}$
$\log_6 16 = 4\log_6 2 = \frac{4(3-a)}{3+a}$

61. $\log_{12}27 = \log_{12}3^3 = 3\log_{12}3$
$\log_{12}3 = (\log_3 12)^{-1}$
$\log_3 12 = 1 + 2\log_3 2 = 1 + 2a$
$\log_{12}3 = \frac{1}{1+2a}$

62. Let $t = 2012!$
$(\log_x t)^{-1} = \log_t x$
Therefore,
$(\log_2 t)^{-1} + (\log_3 t)^{-1} + \ldots + (\log_{2012} t)^{-1}$
$= \log_t 2 + \log_t 3 + \ldots + \log_t 2012$
$= \log_t 2 + \log_t 3 + \ldots + \log_t 2012$
$= \log_t 2\cdot 3\cdot\ldots\cdot 2012$
$= \log_t t$
$= 1$

63. $\log 18 = \log(2\times 3^2) = \log 2 + 2\cdot\log 3 = m + 2n$
$\log 18 = \log_{10}18 = \log_{10}5\log_5 18$
$\log_{10}5 = \log 5 = \log(10\div 2) = 1 - m$
So, $(1 - m)\log_5 18 = m + 2n$
$\log_5 18 = \frac{m+2n}{1-m}$

64. $\frac{1}{32}$

65. $\frac{5}{6}$

66. 2

67. $2e^2$

68. $\frac{e^2}{2}$

69. 125

70. $\frac{4}{3} - 3 = -1\frac{2}{3}$

71. $-5\frac{1}{2}$

72. $\frac{4}{3} - \frac{1}{2} = \frac{5}{6}$

73. $-\ln 2$

74. false

75. false

76. false

77. true

78. false

79. false

80. true

81. true

82. true

83. false
$\log_a (ax)^3 = 3 + 3\log_a x$