Math Power

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Alg I

Exponents and Algebra

Question set [**1 - 4**]

Find the root of each the following.

1. $\sqrt{36x^2}$ $(x > 0)$

2. $\sqrt[3]{\frac{8}{27}}$

3. $\sqrt[5]{-32}$

4. $\sqrt{(-3)^2}$

All the variables represent positive real numbers. Find the root of each the following.

5. $\sqrt{81}$

6. $\sqrt[3]{-27}$

7. $\sqrt[3]{-1}$

8. $\sqrt[4]{(-1)^4}$

9. $\sqrt[5]{(-1)^5}$

Question set [**5 - 19**]

Alg I

10. $\sqrt{100y^2}$

11. $\sqrt[4]{\frac{16}{81}}$

12. $\sqrt[3]{y^6}$

13. $\sqrt{y^6}$

14. $\sqrt[4]{3^8}$

15. $\sqrt{x^2y^2}$

16. $\sqrt[3]{(x+y)^3}$

17. $\sqrt[4]{625y^2}$

18. $\sqrt{(-10)^2}$

19. $\sqrt{\frac{x^2}{y^2}}$

Alg I

Question set [**20 - 23**]

If x is a real number and n is a positive integer ($n > 1$), then

$$x^{\frac{1}{n}} = \sqrt[n]{x}\ .$$

20. $27^{\frac{1}{3}}$

21. $36^{\frac{1}{2}}$

22. $(\frac{1}{16})^{\frac{1}{4}}$

23. $(\frac{9}{25})^{\frac{1}{2}}$

Question set [**24 - 27**]

Simplify each of the following fractional powers.

24. $27^{\frac{2}{3}}$

25. $36^{\frac{3}{2}}$

26. $(\frac{16}{81})^{\frac{3}{4}}$

27. $(\frac{27}{8})^{\frac{4}{3}}$

If x is a real number, m and n are both positive integers, then we have

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

provided that $\sqrt[n]{x}$ is a real number. To extend

the definition of given above to the powers of negative fractional exponents, by observing $-\frac{m}{n} = \frac{-m}{n} = \frac{m}{-n}$, n≠0,
we have

$$x^{-\frac{m}{n}} = \frac{1}{x^{\frac{n}{m}}}$$

Question set [**28 - 31**]
Evaluate the roots each of the following.

28. $25^{\frac{3}{2}}$

29. $16^{\frac{-1}{2}}$

30. $(-27)^{\frac{4}{3}}$

31. $4^{\frac{3}{2}}$

Polynomials: The Basis of Algebra

THEOREM A
[Square Formula]
$(ax + b)^2 = a^2x^2 + 2abx + b^2$
$(ax - b)^2 = a^2x^2 - 2abx + b^2$

Example A:
Multiply with square formula.
(a) $(z + 5)^2 =$

(b) $(9x - 2)^2 =$

Solution:
(a) $z^2 + 2(5)z + 5^2$
$= z^2 + 10z + 25$

(b) $9^2x^2 - 2(9)(2)x + 2^2$
$= 81x^2 - 36x + 4$

Question set [**32 - 40**]
Multiply with square formula.

32. $(4z + 8)^2 =$

33. $(4z - 8)^2 =$

34. $(8 - 4z)^2 =$

35. $(8x + 10)^2 =$

36. $(8x - 10)^2 =$

37. $(-9y - 4)^2 =$

38. $(9y + 4)^2 =$

39. $(3x + 4)^2 =$

40. $(3x + 4y)^2 =$

Question set [**41 - 44**]
Multiply the following polynomials.

41. $(4x + 7)(4x - 7) =$

42. $8(x + 2y)(x - 2y) =$

43. $x(x + 5)(x - 5) =$

44. $2x(x + 1)(x - 1) =$

Example B:
Abstracting the coefficients of a polynomial means leaving out the symbols. The following examples show the representation of a polynomial by abstraction.
(a) $3x^2 - x + 5$

(b) $2x^3 - 5x$

(c) $x^4 - 2x^2 + 1$

Solution:
(a) 3 - 1 + 5

(b) 2 + 0 - 5 + 0
(Note: Leave 0 for missing terms.)

(c) 1 + 0 - 2 + 0 + 1
(Note: Leave 0 for missing terms.)

Question set [**45 - 47**]
Abstract the coefficients of each polynomial.

45. $4x^2 - 3x + 2$

46. $2x^3 - 5x^2 + x - 10$

47. $3x^4 - 2x^3 - 5$

Example C:
Furnishing the polynomial with coefficients means filling a symbol to restore the form of a polynomial.
(a) 3 + 2 - 4 + 0

(b) 4 + 0 - 5 + 0 + 0 - 1

Solution:
(a) $3x^3 + 2x^2 - 4x$

(b) $4x^5 - 5x^3 - 1$

Question set [**48 - 49**]
Furnish the polynomials with the given coefficients.

48. 1 + 2 + 3

49. 1 - 2 + 3 - 4

Example D:
Synthetic multiplication is a method dealing with polynomial multiplication without using symbols. Only the coefficients of the polynomial are retained.
(a) $(3x^2 + 4)(2x - 1)$

(b) $(x^2 + 1)(10x^3 - x - 2)$

Solution:

		3	0	4
	×		2	-1
		-3	0	-4
	6	0	8	
	6	-3	8	-4

(a)
So, we have $(3x^2 + 4)(2x-1)$
$= 6x^3 - 3x^2 + 8x - 4$

		10	0	-1	-2
	×)		1	0	1
		10	0	-1	-2
10	**0**	**-1**	**-2**	0	
10	0	9	-2	-1	-2

(b)
So, $(x^2 + 1)(10x^3 - x - 2)$
$= 10x^5 + 9x^3 - 2x^2 - x - 2$
Note that in a written format, terms with zero-coefficient are usually omitted.

Question set [**50 - 56**]
Multiply the polynomials using synthetic multiplication.

50. $(x^2 + x + 1)(x + 2) =$

51. $(x^2 - 2x + 3)(x - 1) =$

56. $(ab - 5)(ab + 2) =$

52. $(t^2 + 5t - 6)(2t + 1) =$

53. $(x + 2)(x + 3)(x + 4) =$

54. $(5y + 2)(3y + 1) =$

55. $(1 + y)(2 - y) =$

Math Inspiration

57. Three boxes of candy sell for $13.50 by a retailer to gain a profit of 50%. What is the cost of a box of candy?

58. A retailer has some sweaters that cost $28 each. At what price should the sweaters be sold to obtain a profit of 30% of the selling price?

59. A store sells a toy for $53.50, including a 7% sales tax. Determine the price of the toy before tax.

60. After a discount of 20%, a coat was sold at $60. The original price is $60÷_____ = _____.

61. Approximately 60% of body weight is water. Suppose a person weighs 120 lb. About how many pounds is water?

62. Byron purchased a bicycle at a 10% discount sale for $121.50. What was the original price of the bicycle?

63. Byron purchased a bicycle, originally priced $130, at a 10% discount sale. What was the sale price of the bicycle?

64. Daryl, Evan, Fred, and Gerald ran for the school president. After precise counting, there are 500 votes. Daryl got 20% of the votes, and Evan got 6 more percent than Daryl. Gerald would make even with Fred, if Fred could give up 20 votes to him. How many votes did each get?

65. During the summer Rosa earned $950. She saved 40% and spent the rest. How much money did she spend?

66. If a box of candy costs a retailer $2.50 and he wants to make a profit of 50% based on the selling price, what price should he charge for the candy?

67. If cassette tapes are currently $30 each and are expected to increase in price by 20%, what will the new price be?

Question set [**68 - 69**]

If puppy food is currently sold for $30 a pack and are expected to increase by 20% every year.

68. The price will be $30×_____ (decimal) next year.

69. The price will be $30×_____×_____ (decimal) two years from now.

70. In the United States, about 45% of the population wears glasses or contact lenses. How many people would you expect to wear glasses or contact lenses in a group of 800 people?

71. It was reported that a flu epidemic is affecting 6 out of every 15 college students in a certain part of the country. At this rate, how many students will be affected at a university of 15,000 students?

Question set [**72 - 74**]

Jerry made a mistake when he sold a book to a customer. Jerry sold the book 20% higher than the marked price, instead of 20% off. The book was sold for $36.

72. What is the marked price of the book?

73. At what price should the book have been sold?

74. How much should Jerry return to the customer?

75. Jesse used 10 gallons of gasoline to drive 170 miles. How much gasoline will he need to travel 238 miles?

76. Kim pays $630 after tax for a table. If the tax rate is 5%, what is the cost of the table?

77. Louise bought a dress for $140, which represented a 20% discount off the original price. What was the original price of the dress?

78. Mandy bought a 13-inch portable color TV for 20% off of the list price. The list price was $340. What did she pay for the TV?

79. On a certain map, 1 inch represents 15 miles. If two cities are 7 inches apart on the map, find the number of miles between the cities.

80. Pierre bought a coat for $78.00 on a sale at 35% discount. What was the original price?

Math Olympia

81. A man drives from his home at 30 miles per hour to the shopping center which is 20 miles from his home. On the return trip he encounters heavy traffic and averages 12 miles per hour. How much time does the man take in driving to and from the shopping center?

82. In the addition problem at the right, each letter stands for a digit and different letters stand for different digits. What digits do the letters H, E, and A each represent?

```
 HE
+HE
+HE
+HE
----
 AH
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83. The product of two numbers is 144 and their difference is 10. What is the sum of the two numbers?

84. The XYZ club collected a total of $1.21 from its members with each member contributing the same amount. If each member paid for his or her share with 3 coins, how many nickels were contributed?

Answer Key

Exponents and Algebra

1. $6x$
2. $\frac{2}{3}$
3. -2
4. 3
5. 9
6. -3
7. -1
8. 1
9. -1
10. $10y$
11. $\frac{2}{3}$
12. y^2
13. y^3
14. 9
15. xy
16. x+y
17. $5\sqrt{y}$
18. 10
19. x/y
20. $\sqrt[3]{27} = 3$
21. $\sqrt{36} = 6$
22. $\sqrt[4]{\frac{1}{16}} = \frac{1}{2}$
23. $\sqrt{\frac{9}{25}} = \frac{3}{5}$
24. $(27^{\frac{1}{3}})^2 = 9$
25. $(36^{\frac{1}{2}})^3 = 216$
26. $((\frac{16}{81})^{\frac{1}{4}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$
27. $((\frac{27}{8})^{\frac{1}{3}})^4 = (\frac{3}{2})^4 = \frac{81}{16}$
28. $(25^{\frac{1}{2}})^3 = 5^3 = 125$
29. $\frac{1}{16^{\frac{1}{2}}} = \frac{1}{4}$
30. $(-27)^{\frac{4}{3}} = ((-27)^{\frac{1}{3}})^4 = (-3)^4$
31. 8

Polynomials: The Basis of Algebra

32. $16z^2 + 64z + 64$
33. $16z^2 - 64z + 64$
34. $16z^2 - 64z + 64$
35. $64x^2 + 160x + 100$
36. $64x^2 - 160x + 100$
37. $81y^2 + 72y + 16$
38. $81y^2 + 72y + 16$
39. $9x^2 + 24x + 16$
40. $9x^2 + 24xy + 16y^2$
41. $16x^2 - 49$
42. $8x^2 - 32y^2$
43. $x^3 - 25x$
44. $2x^3 - 2x$
45. 4 - 3 + 2

46. 2 - 5 + 1 - 10

47. 3 - 2 + 0 + 0 - 5

48. $x^2 + 2x + 3$

49. $x^3 - 2x^2 + 3x - 4$

50. $x^3 + 3x^2 + 3x + 2$

51. $x^3 - 3x^2 + 5x - 3$

52. $2t^3 + 11t^2 - 7t - 6$

53. $x^3 + 9x^2 + 26x + 24$

54. $15y^2 + 11y + 2$

55. $2 + y - y^2$

56. $a^2b^2 - 3ab - 10$

Math Inspiration

57. 13.5÷3 = 4.50
    1 + 50% = 1.5
    4.5÷1.5 = \$3.00 cost per box

58. 28×(1 + 30%) = 28×1.3 = \$36.40

59. 50×(1 + 7%) = **50**×1.07 = 53.5
    or
    53.5÷1.07 = \$50.00

60. Let the original price be x, then
    $0.8x = 60$
    $x = 60 \div \mathbf{0.8} = \mathbf{\$75}$

61. 120×60% = 120×0.6 = 72 (lb) of water

62. 121.5 ÷ 0.9 = 135

63. 130×0.9 = \$117

64. Daryl: 500×20% = 100
    Evan: 500×26% = 130
    500 - 100 - 130 = 270
    Gerald: 270÷2 - 20 = 115
    Fred: 270÷2 + 20 = 155

65. 950×(1 - 40%) = 950×0.6 = \$570 (saving)

66. 1 + 50% = 1.5
    2.5×1.5 = \$3.75

67. 1 + 20% = 1.2 (after increase)
    30×1.2 = \$36.00

68. \$30×1.2 = \$36

69. \$30×1.2×1.2 = \$43.20

70. 800×45% = 800×0.45 = 360

71. $\frac{6}{15} = \frac{2}{5}$

    $15{,}000 \times \frac{2}{5} = 6{,}000$

72. 1 + 20% = 1.2
    36÷1.2 = \$30.00

73. 1 - 20% = 0.8
    0.80×\$30 = \$24.00

74. 36 - 24 = \$12.00

75. $238 \times \frac{10}{170} = 238 \times \frac{1}{17} = 14$ gallons

76. 630÷(1 + 5%) = 630÷1.05 = \$600

77. 140÷80% = 140÷0.8 = 175

78. 340 × (1 - 20%) = 340×0.8 = \$272

79. 7×15 = 105 (miles)

80. 78÷ 0.65 = \$120

Math Olympia

81. 2 hr 20 min

82. A = 9, E = 3, H = 2

83. 26

84. 22